

The Scaling Relationship for the Length of Tributaries to Lakes

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Key Points:

- The length of tributaries to lakes varies between 0 and 15,000+ kilometers
- Scaling relationships provide simple rules for understanding patterns and variation of river-lake connectivity
- The factors affecting tributary length are: catchment area, lake area, inlet abundance, and junction angle

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Abstract

Globally, the length of tributaries to lakes varies from 0 to more than 15,000 kilometers, but scaling relationships describing this aspect of lake-river connectivity are lacking. In this study, we describe a simple theoretical scaling relationship for tributary length based on the principle of line intercepts of topographic features, and test this theory using data from Scandinavia.

Tributary length increases by 73% for each doubling of lake area. This pattern reflects the relationship between catchment and lake area, and is modified by inlet frequency, junction angle, and lake shape - factors related to specific geologic and hydrologic processes. The theory is precise ($r^2 = 0.74$), with low bias (mean error is 14% of mean tributary length) when the characteristic junction angle ($\sim 76^\circ$) is estimated statistically. Our study bridges the gap between geomorphic and large-scale statistical relationships to provide simple rules for understanding complex patterns of lake-river connectivity.

Plain Language Summary

Patterns of connectivity between lakes and rivers are poorly described because lakes and rivers are typically studied separately. In this study, we develop and test simple rules that describe lake-river connectivity. Specifically, we focus on predicting the length of river tributaries, which varies among lakes from 0 to more than 15,000 kilometers. The most important factors controlling tributary length in our analysis were catchment area, lake area, inlet abundance, and river-lake junction angle. These factors reflect regional climate and bed rock characteristics, lake origin, and catchment geomorphology. Our results connect large-scale statistical patterns with geologic processes, demonstrating how simple rules like those developed in our study can enrich understanding of inland waters.

1 Introduction

Lakes are integral components of many river networks, but research has often focused on rivers or lakes in isolation and not from an integrated perspective that reflects the intimate relationship between these systems (Jones, 2010; Gardner et al., 2019; Richardson et al. 2021). Understanding patterns of connectivity is the first step to understanding the processes that shape river-lake networks, including emergent habitats (e.g., freshwater deltas) and coupled ecosystem characteristics not observed in rivers or lakes alone (Richardson et al. 2021). In particular, there is a need to develop scaling relationships that describe the morphology of lake-river networks. Such relationships provide simple rules for explaining hydrographic patterns, and are widely used to generalize understanding of aquatic systems at regional to global scales (Downing, 2009; Gardner et al., 2019; Seekell et al., 2021a).

Analyses of river-lake networks have typically focused on the influence of lakes on riverine processes. This is probably because the river continuum concept is historically well established and provides a clear basis for interpreting serial discontinuities created by lakes (e.g., Jones, 2010; Gardner et al., 2019; Doretto et al., 2020). However, rivers also have diverse effects on lakes including impact on water balances, the flux of allochthonous nutrients and organic matter, sediment load and the formation of biologically rich delta habitats, and to provide important habitats for many fish species for spawning and growth during juvenile life stages (Richardson et al., 2021). The number of river inlets varies among lakes from zero to several thousand, indicative of a wide range in connectivity patterns and consequent ecosystem impacts (Marcarelli et al., 2019; Richardson et al., 2021; Seekell et al., 2021a). The number of river inlets scales with lake surface area, with scaling coefficients reflecting geologic origin, drainage

density (a function of lithology and climate), and shoreline complexity (Seekell et al. 2021a). However, this is only one aspect of connectivity. The factors controlling other aspects of connectivity, such as tributary length and network structure, are rarely examined even though they may relate to different in-lake characteristics than inlet abundance (cf. Fergus et al., 2017; Lindmark, 2021). For example, while number of inlets may relate to the potential number of deltas in a lake, the length of tributaries is thought to relate to delta size (Richardson et al., 2021). Globally, the length of tributaries varies from 0 to more than 15,000 kilometers, but this aspect of lake-river connectivity is not described by any existing scaling relationships (Supporting Information S1; Gardner et al., 2019; Seekell et al., 2021a).

In this study, we describe a simple theoretical scaling relationship for tributary length to lakes based on the principle of line intercepts of topographic features. We predict how lake and landscape characteristics effect tributary length based on this theoretical scaling relationship. Finally, we test this theory with data from Scandinavian lakes and rivers. Collectively, these analyses provide simple rules that advance understanding of lake-river connectivity at regional and global scales.

2 Theory

The number of river inlets to lakes is primarily a function of lake shore length and landscape drainage density (Seekell et al., 2021a). Specifically, when there are no strong systematic relationships between shorelines and rivers, the expected number of inlets is the shore length divided by the average distance between rivers. This reflects the principle of line intercept of topographic features and is captured by the equation:

$$N = JD_dCA^{D/2}, \quad (1)$$

where N is the number of inlets, D_d is the drainage density (km^{-1}), C describes lake shape (dimensionless), A is the lake surface area (km^2), and D is the fractal dimension of the lake shoreline (Seekell et al. 2021a). This equation is equivalent to the line intercept expectation for number of river inlets because $CA^{D/2}$ is the expected shore length, and D_d is the inverse of the mean orthogonal distance between rivers (Seekell et al. 2021a). The term J (dimensionless) is a correction factor ($J=\sin(\theta \times \pi/180)$) that accounts for variations in the junction angle (θ , degrees) between lakes and rivers. Specifically, the distance between inlets will exceed the orthogonal distance between rivers if rivers intersect lake shorelines at low angles. The correction factor increases the expected distance between streams (reduces the expected number of inlets) for these cases (Seekell et al. 2021a).

The inlet scaling relationship can be extended to identify the key factors effecting the length of tributaries. Specifically, drainage density D_d is the sum of the lengths of all rivers in a catchment (L , km) divided by the catchment area (W , km^2). Substituting L/W for D_d in the inlet-scaling relationship (1), and solving for L , provides a scaling equation for sum of the lengths of all of the lake's tributaries (2).

$$L = \frac{NW}{JCA^{D/2}} \quad (2)$$

Ranges of typical values for these parameters are provided in Table 1.

The scaling equation suggests that tributary length increases with inlet abundance and catchment area, and decreases for lakes with long perimeters (ie. lakes with large surface areas). However, the parameters in the tributary length scaling relationship are not independent. Specifically, both N and W scale sub-linearly by lake area (Nöges, 2009; Walter et al., 2020; Seekell et al., 2021a). Because N scales by area raised half the fractal dimension ($N \sim A^{D/2}$), we

expect that tributary length scales to lake area based on the power exponent relating catchment area to lake area (Seekell et al., 2021a).

Adjustments for junction angle and number of inlets describe variations of the drainage network structure among lakes, and provide the appropriate adjustment for lakes with no river tributaries. The shape factor C describes the influence of lake shape, which controls the relationship between lake area and perimeter. Each of these parameters can be measured directly from maps although, to our knowledge, river-lake junction angles have never been reported (Seekell et al., 2021a). Like other scaling relationships, tributary length scaling is expected to hold as an average among many lakes.

3 Empirical Analysis

3.1 Study Location and Data

We evaluated the tributary length scaling relationship using data from 106 Scandinavian lakes, primarily from the mountainous border region between Sweden and Norway (Table 1). The study lakes and specific methods used to measure features are described in detail by Lindmark (2021) and Seekell et al. (2021a). Briefly, lake surface areas and perimeters were extracted from digitized 1:50,000 scale maps from the Swedish Mapping Agency Lantmäteriet and the Norwegian Water Resource and Energy Directorate. Catchment area was extracted from high-resolution digital elevation models. We counted river inlets and measured stream and river (collectively referred to as rivers) lengths based on map blue lines that represent flowing waters. These matched well to channel networks visible in satellite imagery during cross-validation. Lake-river systems with clear anthropogenic influence, such as dams, were not included in our analysis.

3.2 Data Analysis

First, we evaluated variation and correlation among the parameters in the tributary length scaling relationship. Specifically, we calculated correlation using Spearman's rho correlation coefficient, and variation using the coefficient of variation. We then calculated (no free parameters) the expected tributary length for our study lakes based on the scaling relationship. We evaluated these predictions based on r^2 (squared correlation of predicted versus observed values) as a measure of precision, and mean error as a measure of bias (Seekell et al., 2021b). Our calculations were based on the constant fractal dimension across all lakes ($D/2 = 0.64$), which was previously reported for our study lakes based on the regression of the logarithm of shore length by logarithm of surface area (Seekell et al. 2021a). We also used a constant junction angle correction factor ($J = 2/\pi$) that assumes all angles are equally likely (Seekell et al. 2021a). We did not measure junction angles directly because there is no established method for doing this for lake-river junctions, in particular for small lakes where rivers often intersect highly curved embayments and available methods for measuring riverine junction angles cannot be accurately applied (e.g., Hooshyar et al., 2017). Collectively, these analyses are meant to demonstrate the basic characteristics of the scaling equation, and that the scaling relationship captures real-world patterns of tributary length.

Next, we estimated the average tributary-lake junction angle by regressing (no constant) tributary length by $NW/CA^{D/2}$, and tested if the regression coefficient ($1/J$) is within the plausible boundaries (i.e. $0 < J \leq 1$). We also examined if this provides a better fit than the assumption that all angles are equally likely, which is used in our deterministic calculations. We expected

high junction angles (i.e. close to 90°) based on maps, satellite image, and our field experience in the study region (e.g., Norman et al., 2022; Klaus et al., 2021; Seekell et al., 2021b; Riley & Seekell, 2021). This is not a trivial expectation because qualitative evidence indicates that there is a substantial range of junction angles, including very low junction angles in some regions (Supporting Information S2). We fit the parameter using non-linear least squares, which is an appropriate approach for linear models with interval constraints such as the physical constraints on junction angle (Table 1; cf. Poi, 2008). Compared to a simple linear regression, using nonlinear least squares resulted in no meaningful difference in junction angle estimate, or measures of precision and bias, but does create differences in 95% confidence intervals which are constrained within the physical limits for the nonlinear least squares analysis but not for a simple linear regression.

Finally, we tested if the scaling exponent between tributary length and lake area was equivalent to that for catchment to lake area by evaluating the 95% confidence intervals for the two scaling relationships. We expected no significant difference based on the structure of the theoretical scaling relationship. We fit the relationships by ordinary least squares to \log_{10} transformed variables. Lakes without tributaries were not included in the tributary length-lake area relationship.

Our analysis was implemented using R version 4.0.2 with the CAR package (Fox & Weisberg, 2019; R Core Team, 2020). We report confidence intervals based on percentiles from bootstrapping ($n = 9,999$).

4 Results

Tributary length varies substantially among the study lakes, and is strongly correlated with catchment area and inlet abundance (Table 2). There is a moderate correlation between tributary length and lake area. Additionally, lake area is correlated with catchment area and inlet abundance. These results are consistent with our predictions based on the functional form of the scaling relationship and previous empirical analyses. The shape factor varies the least among lakes and is not correlated with tributary length or the other parameters in the scaling relationship (Table 2).

The scaling relationship captured the basic patterns of tributary length with relatively precise predictions ($r^2=0.74$) when assuming that all junction angles are equally likely. This is demonstrated in Figure 1A, where the points are distributed along a 1:1 line representing the observed values. However, the fit is biased with the mean error of 6.8 km, which is 39% of the mean tributary length (ie. most of the points are above the 1:1 line instead of being evenly distributed above and below the 1:1 line).

The scaling relationship had improved fit when the characteristic junction angle was fit statistically (Figure 1B). Specifically, there is no material difference in precision ($r^2 = 0.74$), but bias is reduced substantially (mean error = 2.5 km, which is 14% of the mean tributary length). The characteristic junction angle was 76° . The 95% confidence interval was wide, 95% CI = 35° - 90° , but was not symmetrically distributed because of the physical limits of this parameter. Almost all (90.7%) of bootstrapped estimates were greater than 45° , and the median of the bootstrapped estimates was 76° . Overall, the results are consistent with our expectation of a relatively high junction angle for the study region.

Catchment area scales sub-linearly by lake area (Figure 2A). Hence, the average catchment to lake area ratio is lower for larger lakes than smaller lakes. These patterns are consistent with previous reports from other physiographic regions (Nöges, 2009; Walter et al.,

2020). Tributary length also scales by lake area, although by a somewhat lower but not significantly different exponent compared to catchment area (Figure 2B). This pattern is consistent with our expectations developed based on the functional form of the tributary length scaling relationship, and correlations between area, inlet abundance, and catchment area.

5 Discussion

Scaling relationships provide simple rules for understanding hydrographic patterns at regional and global scales. Our study contributes to this understanding by identifying the basic factors that are related to variation in tributary length among lakes. Prior research has typically focused on rivers or lakes in isolation, whereas our study provides an integrated perspective that reflects the close relationship between these systems. Catchment area, lake area, inlet abundance, lake-river junction angle, and lake shape are the primary factors determining the variation in tributary length among lakes, with catchment area and lake shape having the highest and lowest importance, respectively. These factors relate to specific geological processes, and hence our results can be the basis for developing new testable hypotheses about the connection between physical processes and the large-scale statistical relationships that emerge in global analyses of lake characteristics.

Catchment area is the dominant factor creating variation in tributary length among lakes due to its high variability and their strong correlation. Larger catchments have longer total tributary length which is intuitive because catchment areas vary among lakes by at least six orders of magnitude, but drainage density (tributary length divided by catchment area) is relatively constant within regions (Lapierre et al., 2015; Schneider et al., 2017; Lapierre et al., 2018; Walter et al., 2020). The variation of lake catchment sizes probably originates from the approximate scale-invariant nature of Earth's topography, which is associated with highly skewed size-distributions, and itself emerges from fracture and deformation processes at different scales, although to our knowledge this has not been explicitly studied for lake catchments and therefore is somewhat speculative (Malinverno, 1995; Hinkle et al., 2020).

The number of inlets and junction angle are the second most important factors related to tributary length based on its own moderate variation and their strong correlation. Number of inlets is a particularly interesting characteristic because, in addition to tributary length, it relates to the potential for freshwater deltas – emergent habitats that primarily exist at intersection points in combined river-lake networks (Richardson et al., 2021). We expect that these factors vary regionally, and that this variation will facilitate the application of our tributary length scaling relationship to other regions. Specifically, inlet abundance and tributary length both reflect drainage density, which varies among regions due to climate and lithology (Schneider et al., 2017). Lakes in arid regions with low drainage density should have fewer inlets and less tributary length than lakes in humid regions, at least when drainage density is calculated based on permanently flowing rivers. Junction angle was an important factor for accurate prediction of lake-river connectivity in both the present study and a previous study focused on inlet abundance (Seekell et al., 2021a). However, little is known about lake-river junction angles beyond that surficial geomorphology sometimes creates variation in this parameter. For example, the presence of transverse dunes in the vicinity of lake shorelines can direct rivers to intersect lakes at low angles ($< 45^\circ$) (Supporting Information S2). Freshwater dunes exist in the lake-rich region around the Laurentian Great Lakes, but not in the lake-rich Scandinavian mountains where our study lakes are located, and hence regional-scale variation in junction angle may exist due to the presence and absence of these dunes (Martínez et al., 2004). It is remarkable that our estimate of

characteristic river-lake junction angle $\approx 76^\circ$ is very similar to the characteristic junction angle for flow-dominated river channels ($\approx 75^\circ$) (Hooshyar et al., 2017). When looking across climate gradients, our measurement is also very similar to the characteristic junction angle for rivers in humid regions ($\approx 72^\circ$) (Seybold et al., 2017). The extent to which existing junction angle theories for river networks translate to river-lake networks is unknown, although the observation of low lake-river junction angles in the vicinity of the Laurentian Great Lakes indicates that the characteristic junction angle in our study region is not universal.

Tributary length was correlated with lake surface area. Lake area has a strongly skewed distribution - there are many small lakes and few large lakes - which implies that there are many lakes with no or short tributary networks, and few lakes with very long tributary networks. The relationship between tributary length and lake area is weaker than the relationship of tributary length to catchment area. Specifically, the relationship between tributary length and lake area arises because catchment area and lake area are positively correlated. While it is sensible that larger lakes should have larger catchments than smaller lakes (i.e. because larger lakes stand to capture water from what would be otherwise divided catchments), empirical patterns for diverse physiographic regions are not strong. For example, previously reported scaling relationships for catchment and lake area from the United States, New Zealand, and Europe demonstrate that catchment area often varies by more than three orders of magnitude for lakes of the same size (Nöges, 2009; Walter et al. 2020). Similar variation was evident in our analysis (e.g., Figure 2A). The broader landscape is shaped by large-scale processes acting somewhat uniformly over an extended period of time, whereas lake formation is often a catastrophic change within the landscape (Timms, 1992). Hence, the factors determining catchment size and lake sizes are either not closely linked, or at least have substantial stochastic components, such that scaling relationships for tributary length by lake area will probably always be characterized by substantial residual variation.

Lake shape factor has little influence on tributary length. This is probably because lakes and rivers have different geologic origins. Lake shape reflects specific originating processes, for example volcanic crater lakes are sub-circular, whereas glacial scour lakes have highly irregular shapes (Seekell et al., 2021a). Rivers form when topographic irregularities lead to preferential overland and subsurface flow, from which channels evolve over time through erosion (Wohl, 2009). The lack of a clear systematic relationship between lake shape and tributary length is consistent with the assumptions used to derive the tributary length scaling relationship. While lake shape explains little variation in tributary length, it remains a necessary factor within the scaling relationship to ensure that it is dimensionally correct.

Most of Earth's lakes are small ($< 10 \text{ km}^2$), are located in the high northern latitudes, and were formed by glacial processes, similar to the lakes in our empirical analysis (Meybeck, 1995; Verpoorter et al., 2014). Hence, our results are likely generalizable to a large proportion of lakes. However, our results may not be generalizable to all lakes with anthropogenic origins (i.e. reservoirs). For example, reservoirs formed by damming large rivers may deviate from our scaling relationship because the assumption of weak systematic relationships between rivers and shorelines is not met. Our study has two additional limitations, both related to the measurement of river length. First, river length measurements are scale dependent, with higher-resolution maps producing longer length estimates than lower-resolution maps (Tarboton et al., 1988). Our study minimizes this limitation by only considering lakes and rivers mapped at the same scale. Second, many small rivers are ephemeral and are sometimes omitted from maps (e.g., Downing et al., 2012). While our analysis is based on map blue lines, which represent permanent channels,

they accurately represented channels visible in satellite imagery and when calculating flow lines from digital elevation models, indicating that potential ephemeral streams are unlikely to have a material impact on our empirical analysis.

6 Conclusions

The goal of developing scaling relationships is to provide simple rules that capture the essence of hydrographic patterns at large scales. Scaling relationships also serve as null-hypotheses against special-case systems can be evaluated, providing context for developing and testing detailed hypotheses about systems that deviate from overall patterns (cf. Goodchild, 1988). Our study describes basic rules for understanding how lake, river, and landscape factors influence the length of river tributaries to lakes. In particular, tributary length primarily reflects catchment area and inlet abundance, with junction angle, lake area, and lake shape having secondary influence. Our analysis highlights several gaps in the understanding of lake morphometry and lake-river connectivity within broader hydrologic networks. In particular, relatively little is known about lake-river junction angles and the factors creating variation in lake catchment size. We made the first estimate of river-lake junction angle and found it approximately equal to the characteristic junction angle for flow-dominated river channels. Overall, our study both advances the basic understanding of the factors constraining lake-river connectivity and delineates an agenda for future research on this topic.

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Open Research

The data used in this study are archived on Zenodo at doi: <https://doi.org/10.5281/zenodo.4612170>. The previously published data informing values in Table 1 are available from their original sources, specifically: Mark (1983), Seekell et al., (2014), Lapierre et al. (2015), Cael and Seekell (2016), Walter et al. (2020), and Seekell et al. (2021a). Previously published data used in the Supplemental Materials are available from their original sources, specifically: Lehner and Döll (2004) and Lehner and Grill (2013). The code for our statistical analysis is archived on Zenodo at doi: [10.5281/zenodo.6370036](https://doi.org/10.5281/zenodo.6370036).

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Figure Legends

Figure 1. A) Predicted versus observed tributary lengths when the predicted length is calculated deterministically (no free parameters) assuming that all river-lake junction angles are equally likely. B) Predicted versus observed tributary lengths when the predicted length is calculated with characteristic junction angle as a free-parameter that is estimated statistically. In both panels, the patterns are shown on \log_{10} axes to emphasize the difference between deterministic and statistical fit, but the statistical fitting and measures of bias and precision are based on the original units (as opposed to \log_{10} units). The inset figures display the data and relationships on the original scale.

Figure 2. A) The scaling relationship between catchment and lake areas for the study lakes. The intercept for the relationship is 17.8 km^2 . B) The scaling relationship between tributary length and lake area. In intercept for the relationship is 22.9 km . The confidence intervals for the two relationships overlap, suggesting no significant difference in the scaling exponents.

Table 1. Typical values and physical limits for parameters in the tributary length scaling relationship, as well as the median and range of values from the study lakes. Drainage ratio is catchment area divided by lake area. Sources provided are for the typical values and physical limits.

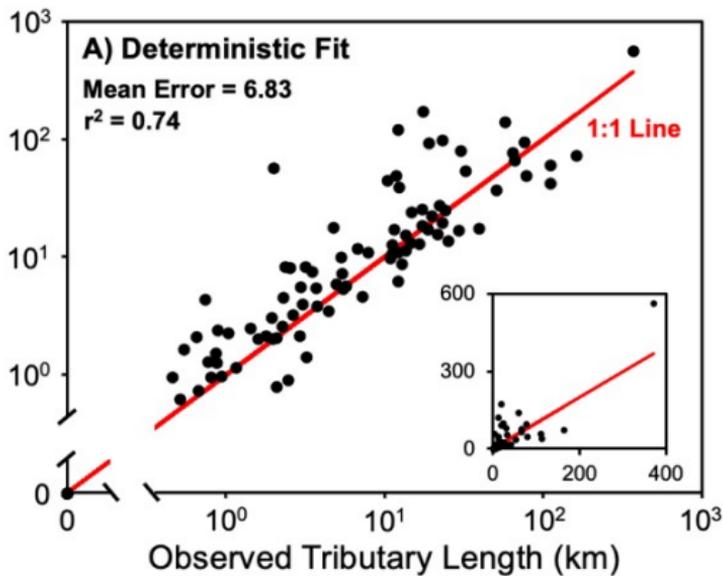
Parameter	Typical value	Physical Limits	Median in this study	Range in this study	Source
Inlet Abundance (N)	0-100's	≥ 0	2	0-26	Mark (1983); Seekell et al. (2021a)
Catchment area (W)*	0.01-10 km ²	$0 < A < 5.1 \times 10^8$ km ²	3.32 km ²	0.05-232 km ²	Lapierre et al. (2015); Walter et al. (2020)
Shape factor (C)	4-12	$\geq 2\pi^{0.5}$	7.99	4.21-17.02	Seekell et al. (2021a)
Lake surface area (A)*	0.01-10 km ²	$0 < A < 5.1 \times 10^8$ km ²	0.14 km ²	0.01-3.78 km ²	Cael and Seekell (2016)
Fractal Dimension (D)	1.28	$1 \leq D < 2$	1.28	---	Seekell et al. (2021a)
Junction Angle (θ)	45-90°	$0 \leq \theta \leq 90^\circ$	---	---	Seekell et al. (2021a); This study
Tributary Length (L)	0-100's km	≥ 0	3.71 km	0-371 km	This study
Drainage Ratio (W/A)	1-100	> 0	20.23	2.3-358	Seekell et al. (2014); Walter et al. (2020)

* The maximum limit given is Earth's surface area.

Table 2. The coefficient of variation (CV) and Spearman's rho correlation matrix for parameters in the tributary length scaling relationships.

	CV	Correlation matrix			
		Lake Area (A)	Catchment Area (W)	No. Inlets (N)	Shape Factor (C)
Tributary Length (L)	2.47	0.66	0.91	0.85	0.02
Lake Area (A)	1.66	---	0.68	0.72	-0.08
Catchment Area (W)	2.44	0.68	---	0.76	0.00
No. Inlets (N)	1.28	0.72	0.76	---	0.06
Shape Factor (C)	0.27	-0.08	0.00	0.06	---

Predicted
Tributary Length
(km)



Predicted
Tributary Length
(km)

